Summary of Work Done until 7/30/2014

In this document I present a relatively brief discussion of the models and results I've gotten so far. I'll divide it into sections for clarity, and present as many graphs/results as possible. If you have any questions, let me know.

Simple Bernoulli Model

Based on an idea given by Luca after one of my first weekly reports, I decided to see if I could make a plot of the flow rate as a function of the pressure difference between the sphere and the cryostat with constant xenon properties. The calculation is based on a simple Bernoulli analysis with the friction terms added:

$$\frac{P_{cryostat}}{\rho} + gh_{cryostat} = \frac{P_{sphere}}{\rho} + gh_{sphere} + \frac{v^2}{2} + \frac{P_{loss}}{\rho}$$

Where ρ is the density of xenon, P is the pressure of the component in subscript (with P_{loss} being the pressure loss due to friction), h is the height of the component in subscript relative to the floor, g is the acceleration of gravity, and v is the mean velocity of the flow. The v is the variable we are solving for, and it is transformed into the mass flow rate post-calculation by $= \rho A v$, where A is the cross-sectional area of the flow.

The properties of xenon were given to me by Luca in the same email chain for that weekly report; 2978 kg/m^3 as the density of xenon, and the height difference between the cryostat and the sphere being 3.355 m. The length of the pipe was approximated based on Donato's drawings and Jean-Marie's drawings to approximately 13 m, and the internal diameter across the whole fast recovery pipe was approximated as 13.32 mm. (This is technically incorrect, as Donato's drawings suggest a slightly larger pipe diameter in the cryostat to cryogenics section; however, the change is minimal enough to be negligible.)

Reordering for the velocity, we get:

$$v = \sqrt{2\left[\frac{\left(P_{cryostat} - P_{sphere}\right)}{\rho} + g\left(h_{cryostat} - h_{sphere}\right) - \frac{P_{loss}}{\rho}\right]}$$

This pressure loss term is the main source of complexity in the calculation; as I demonstrate soon, this equation becomes a set of two nested implicit equations.

The pressure losses due to friction, which are the main source of pressure losses in general, are determined by the Darcy-Weisbach equation:

$$P_{loss} = f \frac{L}{D} \frac{\rho v^2}{2}$$

Where f is the dimensionless Darcy friction factor, L is the total length of the pipe, and D is the internal diameter of the pipe. This friction factor is particularly difficult to calculate in the turbulent flow regime, which is almost always the type of flow in our system. The Colebrook equation, which is implicit in f, is the best and most applicable way to calculate it. This equation is:

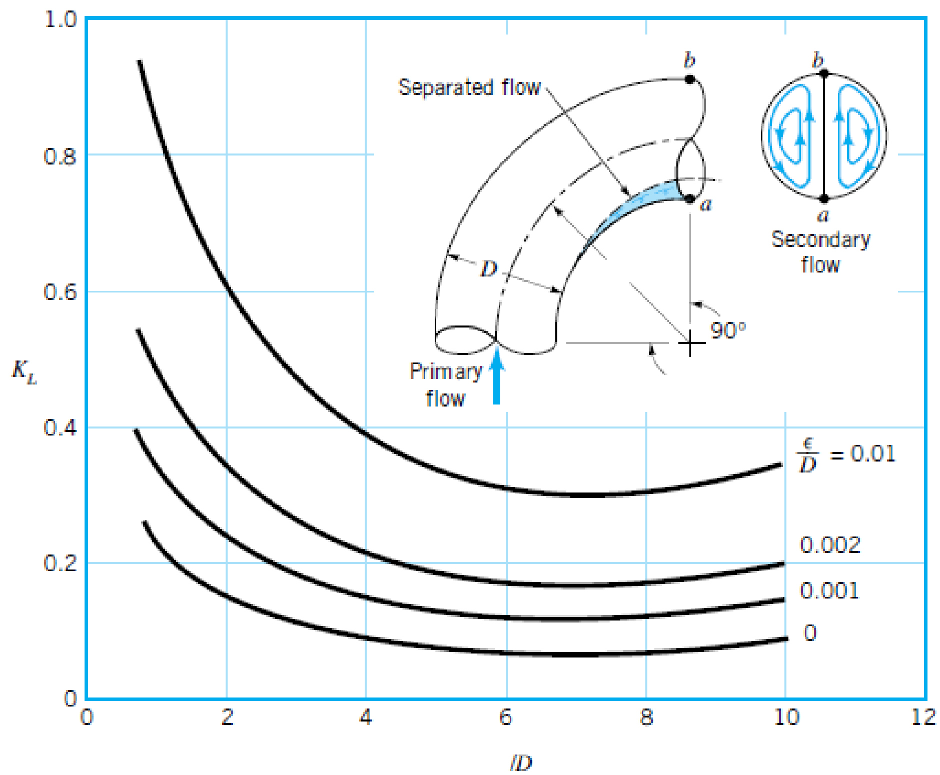
$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}} \right)$$

Where ε is the roughness of the pipe, and Re is the dimensionless Reynolds number of the flow. In pipe flow, this is equal to $Re = \frac{\rho v D}{\mu}$. ($\mu$ is the viscosity of xenon.) In our particular case, ε is always zero since the interior of the fast recovery line is electropolished. Notice how this friction factor is both implicitly calculated and also relies on the velocity; this characteristic causes the total calculation to be a doubly-nested implicit equation.

The losses due to bending in the geometry are calculated with the following equation:

$$\Delta P = K \frac{\rho v^2}{2}$$

Where K is a minor loss coefficient that depends on the geometry of the bend. Taking the 85 degree bends as 90 degree bends, the K factor is calculated using the following graph, found in this resource: http://www.cs.cdu.edu.au/homepages/jmitroy/eng247/sect10.pdf. Note the bottom label is missing an R; the x-axis variable should be the radius of curvature over the pipe diameter.



For the valve losses, Jean-Marie provided a schematic with the flow coefficient of the valve in the ReStoX section of the line, which is $C_v = 4.2$ US gallons/minute. The model for calculating the pressure loss in the valve is;

$$\Delta P = SG\frac{A^2v^2}{C_v^2}$$

Where SG is the specific gravity of xenon and A is the cross-sectional area of the pipe. Be careful; this gives the pressure drop in psi, and the result should be multiplied by the correct conversion factor in order to be correct.

The equation now becomes the following:

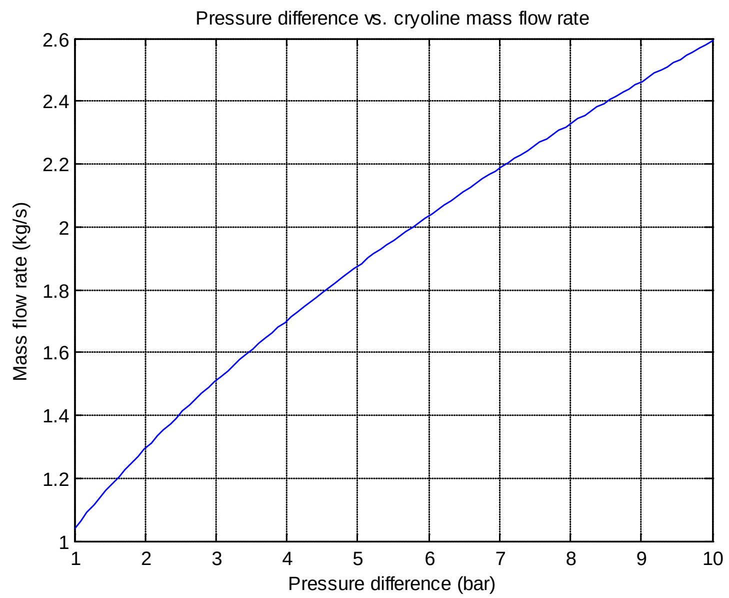
$$v = \sqrt{2\left[\frac{(P_{cryostat} - P_{sphere})}{\rho} + g(h_{cryostat} - h_{sphere}) - f\frac{L}{D}\frac{v^2}{2} - K\frac{v^2}{2} - \frac{SG}{\rho}\frac{Av^2}{C_v^2}\right]}$$

Where f is the calculated with:

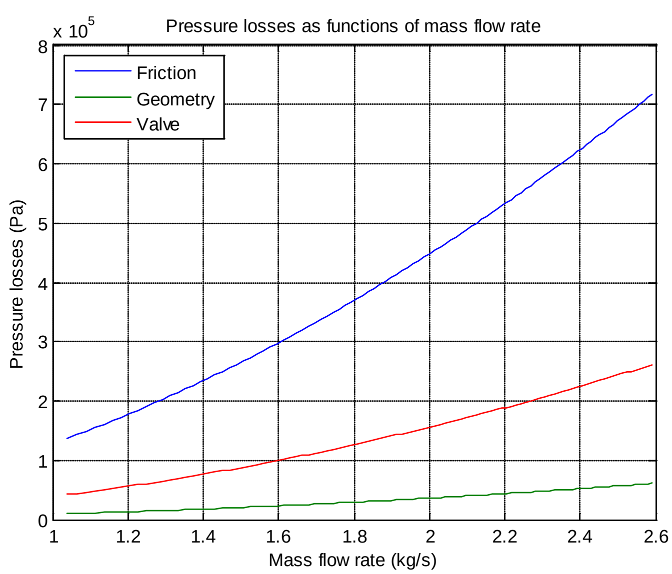
$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\frac{\rho v D}{\mu}\sqrt{f}}\right)$$

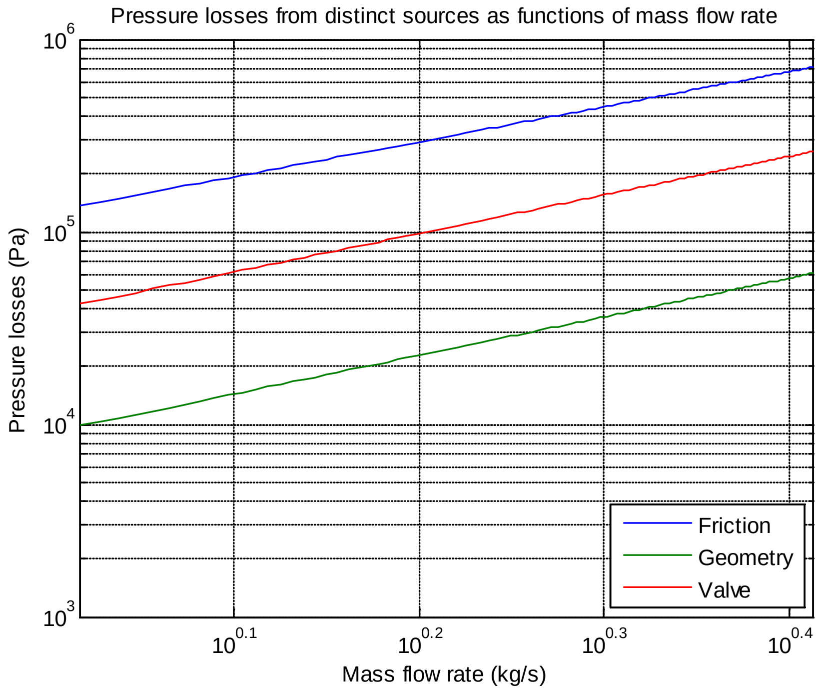
These are the nested implicit equations I had mentioned before. I use a series of suitable nested optimization algorithms to calculate the correct velocity (and the correct pressure losses) by correlation. In short, I use trial velocities to calculate the pressure losses in the pipe, and then choose the trial velocity that outputs itself when its associated pressure losses are put into the Bernoulli equation above.

Below is a plot the mass flow rate as a function of the pressure difference between the cryostat and the sphere. This flow rate is good for our intention of quick recovery; and I have checked and re-checked that the calculation is correct. (Luca's program agrees with mine, albeit without friction.)



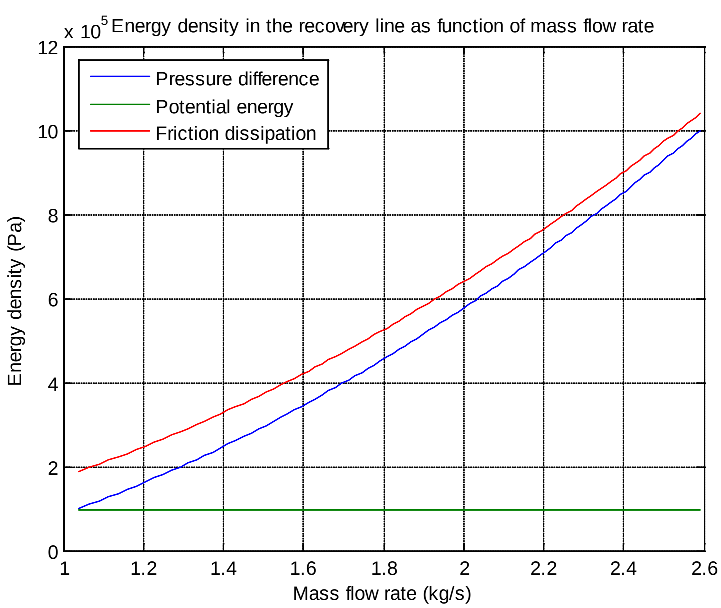
In order to visualize the magnitude of the friction sources relative to each other, I plotted the pressure losses due to the different friction sources as functions of the flow rate on a normal plot and a log-log plot. (I include these below.)





Just like Jean-Marie suggested, the valve losses are much higher than the total geometry losses, with friction being the largest source of loss. The log-log plot makes it clear that all of the losses are going up with the same exponent, which makes sense since they all go up with the square of the flow rate/velocity according to their respective models (except for the slight nonlinearity in the friction one.)

As Luca suggested, I also made a plot describing the different specific energies in the flow system as a function of the flow rate for clarity. Don't be alarmed; even though the friction losses are higher than the pressure difference, the addition of the potential energy factor is what gives us a positive flow.



Jean-Marie's order-of-magnitude estimates for the friction losses agree with mine, so I have no problem doubting these results either.

The natural next step of progression is to consider/include the evaporation of xenon in the pipe; at Jean-Marie's suggestion, I performed an altered version of the above calculation at a cryostat pressure of 3.2 bar and a sphere pressure of 0.9 bar (both being at the saturation temperatures of those pressures).

Simple Bernoulli model with evaporation

Knowing the initial and final states, we can immediately calculate how much liquid xenon evaporates in the pipe:

$$\chi = \frac{h_u^L - h_d^L}{h_d^V - h_d^L}$$

χ is the weight fraction of liquid vaporized, h_u^L is the enthalpy of the liquid xenon before the pressure drop, h_d^L is the enthalpy of the remaining liquid xenon after the pressure drop and h_d^V is the xenon vapor enthalpy (naturally after the pressure drop).

With the liquid/gaseous xenon proportion, I can calculate the xenon density and viscosity after the pressure drop. (I use liquid xenon data at the cryostat pressure/temperature for the initial state, and a simplified ideal solution model for the viscosity calculation.)

$$\frac{1}{\rho_d} = \frac{1}{\rho_v}\chi + \frac{1}{\rho_l}(1-x)$$
$$\frac{1}{\mu_d} = \frac{1}{\mu_v}\chi + \frac{1}{\mu_l}(1-x)$$

ρ_d is the "average" density at the final state, $\mu_d$is the "average" viscosity at the final state, ρ_v is the xenon vapor density at the final state, $\mu_v$is the vapor viscosity at the final state, μ_l is the liquid xenon viscosity at the final state and ρ_l is the liquid xenon density at the final state.

Having this, I calculate an average viscosity and density in the pipe for approximate Reynolds number calculations assuming a linear pressure drop in the pipe; therefore, the average density and viscosity is determined from the "mean" pressure and weight fraction in the pipe (2.05 bar and 4.5% weight fraction).

Afterwards, I can use the same method I used before in the simple analysis to calculate the velocity/flow rate, albeit with a few changes to the governing equation:

$$P_{cryostat} + \rho_u g h_{cryostat} + \frac{\rho_u v_{entrance}^2}{2} = P_{sphere} + \rho_d g h_{sphere} + \frac{\rho_d v_{exit}^2}{2} + P_{loss}$$

Since the flow rate in the pipe needs to be constant, $\rho_u v_{entrance} = \rho_d v_{exit}$, which gives us an expression for the entrance velocity as a simple function of the exit velocity.

Rewriting the Bernoulli equation in terms of the exit velocity (naturally still implicit due to the loss term):

$$v_{exit} = \sqrt{\frac{2}{1-\frac{\rho_u}{\rho_d}}\left[\frac{\Delta P}{\rho_d} + g\left(\frac{\rho_u}{\rho_d} h_{cryostat} - h_{sphere}\right) - \frac{P_{loss}}{\rho_d}\right]}$$

At this point, I use two different models for the average pressure losses in the pipe. One assumes the average density and viscosity of the xenon in the flow is an average of the entrance and exit states; the other assumes the average density and viscosities are the properties at the average state (2.05 bar and a 4.5% weight fraction.) These are their corresponding equations:

$$\rho_{a_1} = \frac{\rho_u + \rho_d}{2},\ \mu_{avg_1} = \frac{\mu_u + \mu_d}{2}$$

$$\rho_{a_2} = \rho\left(\frac{P_{cryostat} + P_{sphere}}{2}, \frac{\chi}{2}\right),\ \mu_{avg_2} = \mu\left(\frac{P_{cryostat} + P_{sphere}}{2}, \frac{\chi}{2}\right)$$

The velocity and Reynolds number formula for use in the pressure loss calculations in both models are the following;

$$v_a = \frac{\rho_d}{\rho_a} v_{exit}$$

$$Re = \frac{\rho_a v_a D}{\mu_a}$$

The mass flow rate is then calculated by simply using $Q = \rho_d v_{exit} A = \rho_d v_{exit} \frac{D^2}{4}\pi$. The nature of the way the velocity is calculated ensures that the mass flow rate remains constant inside of the pipe.

With these considerations and in the current version of the program, I am obtaining the following results;

1) For the first model using the mean density, I obtain a mass flow rate of 0.7308 kg/s and an exit velocity of 54.1932 m/s. This is equivalent to a recuperation time of 1 hour and 8 minutes.
2) For the second model using the density at the mean state, I obtain a mass flow rate of 0.3123 kg/s and an exit velocity of 23.1619 m/s. This is equivalent to a recuperation time of 2 hours and 40 minutes.

As you can see, the model for what the average properties are in the pipe changes our operation parameters drastically. Jean-Marie and I both agree that the second model is the one closer to reality; however, it's evident that only a differential analysis that traces the pressure drop in the pipe can correctly tell us what the steady-state operation parameters are. This algorithm I describe below.

Differential model

The differential model, which I assume to be both the final and the most precise model, sets up a series of nodes along the streamline coordinates of the pipe and then performs a series of finite-difference calculations between them in order to calculate the thermodynamic state and operating conditions at every node. The process is the following;

1) Initial conditions and geometry considerations are defined. For the geometry, based on Donato's drawings, Guillaume's descriptions and Jean-Marie's drawings, I have the following geometric specifications. Sections a through d have an internal diameter of 16 mm, e through g an internal diameter of 15.748 mm, and everything afterwards an internal diameter of 13.32 mm.
    a. 2651.6 mm of tubing inside the cryostat itself. I've been told this tube is flexible and the way it's going to be set up is still to be determined, so I just assume it goes straight down (even though I know it doesn't).
    b. 908.1 mm of tubing from the cryostat going straight up.
    c. 556.3237 mm of tubing in the large bend leading out of the water tank.
    d. 5603.5 mm of tubing at an inclination of 5 degrees leading into Guillaume's cryogenics.
    e. 776.9 mm of pipe inside the cryogenics still going 5 degrees up.
    f. 59.847 mm of pipe bending 90 degrees towards the right.
    g. 421.9 mm of horizontal pipe leading out of the cryogenics.
    h. 412.574 mm of horizontal pipe outside of the cryogenics.
    i. 56.55 mm of pipe bending downwards to a 5 degree slope.
    j. 1131.55 mm of pipe going downwards at a 5 degree slope.
    k. 56.55 mm of pipe bending to go straight down.
    l. 4625.107 mm of pipe going straight down.
    m. 56.55 mm of pipe bending 90 degrees.
    n. 2020.321 mm of horizontal pipe.
    o. 56.55 mm of pipe bending 90 degrees.
    p. 853 mm of pipe of horizontal pipe.
    q. 56.55 of pipe bending 90 degrees downwards.
    r. 679.5 mm of pipe going straight down into the ReStoX sphere.

    This gives us a grand total of 20.9829 m of fast recovery tubing. The thermo-fluidic properties of the xenon are calculated from interpolation of values extracted from NIST based on the initial state of 3.2 bar at saturation temperature in the cryostat and a vapor quality (gas weight fraction) of 0.
2) A trial "inlet" velocity is selected and applied to the initial node. With this, the mass flow rate (which remains constant in the pipe) is calculated using the initial density and cross-section area.
3) The pressure drop or gain due to altitude is calculated between nodes using $\Delta P_{height} = \rho g l$, where ρ is the density of xenon at the first node, l is the length between nodes, and g is a

function of l to correctly model the angled sections of piping as well as the sign of the pressure drop/gain.

4) The pressure loss due to friction between nodes is calculated using $\Delta P_{friction} = f\frac{l}{D}\frac{\rho v^2}{2}$, where D is the diameter of the pipe at the first node and v is the node's corresponding velocity. The friction factor is calculated using the same implicit Colebrook formula mentioned in the previous sections.
5) The pressure loss due to bending or sudden pipe contraction between nodes is calculated using $\Delta P_{geometry} = K\frac{\rho v^2}{2}\frac{l}{L_{geom}}$, where K is a loss coefficient determined with the resource mentioned above and L_{geom} is the length of the geometric object in question. These bending losses only "activate" when the program detects it is analyzing a geometrically non-standard section of piping; since the load would be applied $\frac{L_{geom}}{l}$ times, the total pressure loss from the geometry would be $K\frac{\rho v^2}{2}$, matching the correct value.
6) Similarly, the pressure loss due to the valve is calculated using $\Delta P_{valve} = \frac{l}{L_{geom}}\frac{\rho}{\rho_{water}}\frac{A^2v^2}{C_v^2}$ and only "activates" when the program detects a part of the pipe in the valve is being analyzed.
7) The pressure of the next node is calculated using $P_{next} = P_{previous} + \Delta P_{height} - \Delta P_{friction} - \Delta P_{geometry} - \Delta P_{valve}$. With this, the thermophysical properties of the liquid and gaseous xenon at the new pressure are extracted with interpolation from the uploaded NIST tables.
8) Since we approximate the flash evaporation in the pipe as an isenthalpic process, we can immediately calculate the new weight fraction of gaseous xenon at the next node using the following equation:

$$\chi_{next} = \frac{\chi h_v + (1-\chi)h_l - h_l^{new}}{h_v^{new} - h_l^{new}}$$

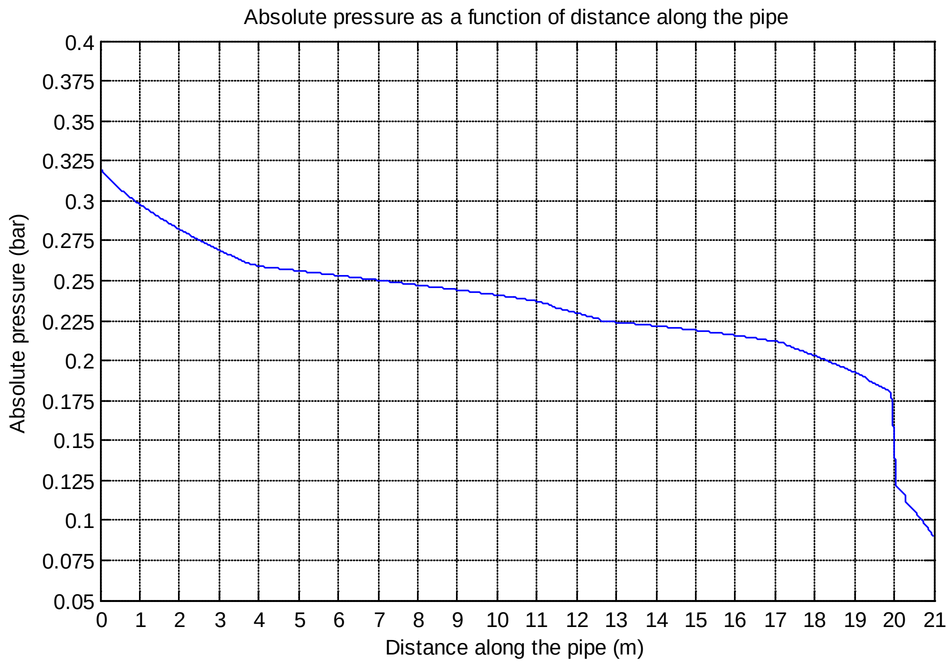
9) With the new weight fraction, the "average", or true, density and viscosity of xenon at the next node are calculated using the following equations:

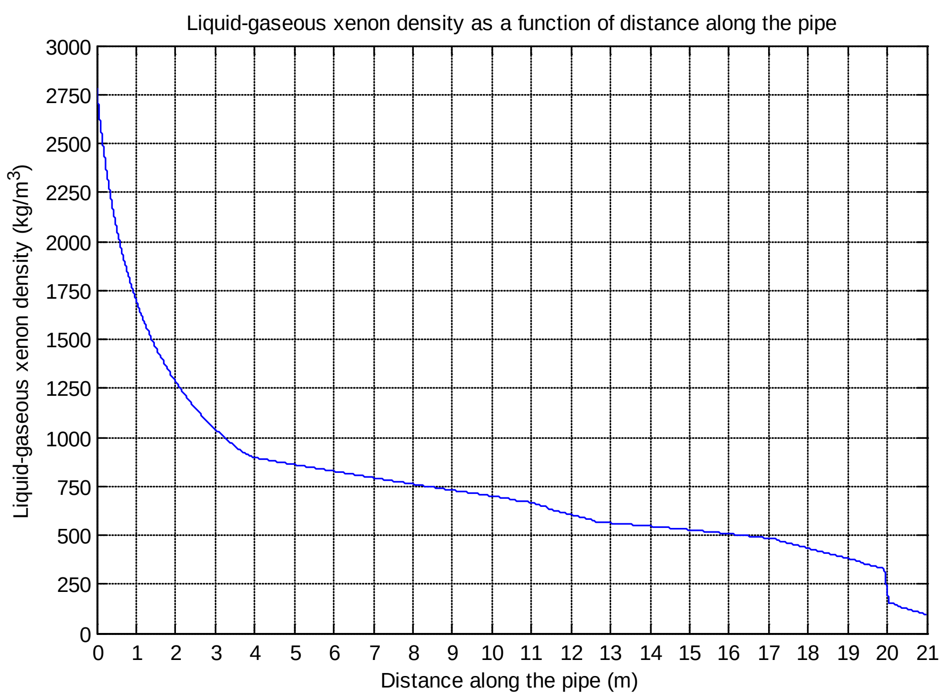
$$\frac{1}{\rho_{new}} = \frac{1}{\rho_v^{new}}\chi_{new} + \frac{1}{\rho_l^{new}}(1 - x_{new})$$

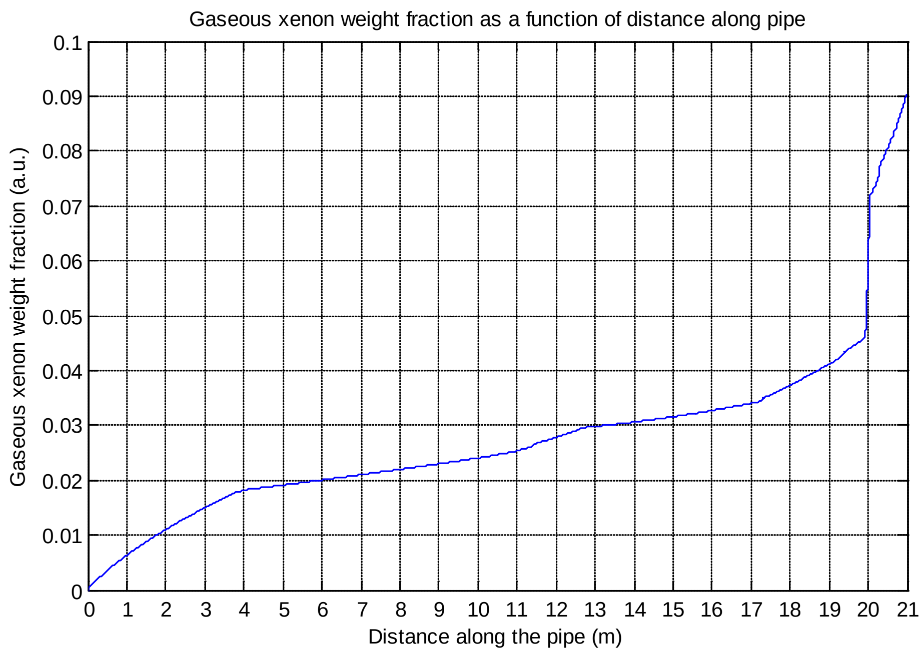
$$\frac{1}{\mu_{new}} = \frac{1}{\mu_v^{new}}\chi_{new} + \frac{1}{\mu_l^{new}}(1 - x_{new})$$

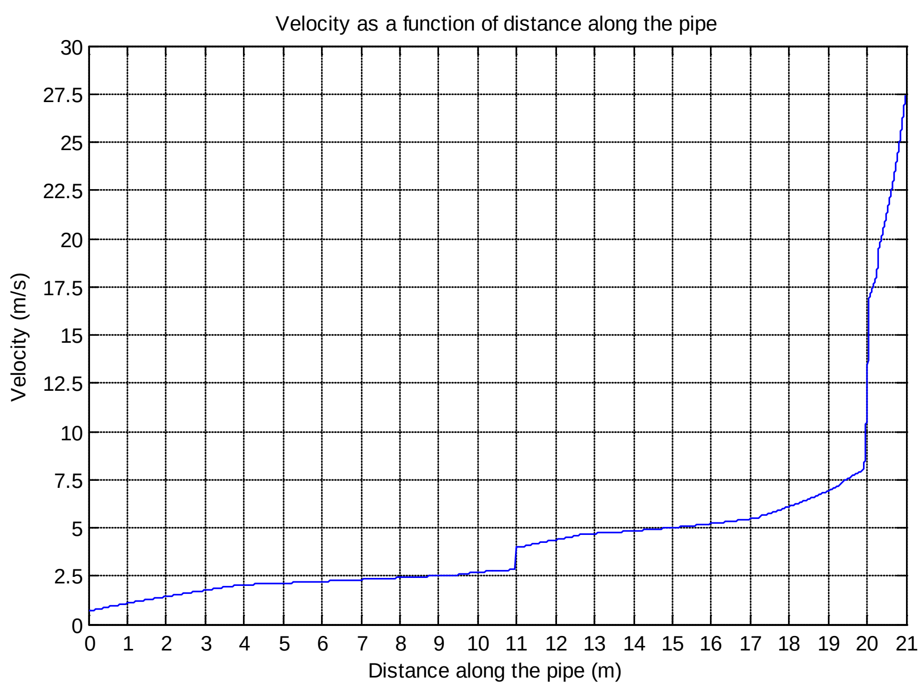
10) Since the mass flow rate must remain constant, the new velocity is calculated exploiting this by using $v_{new} = \frac{Q}{\rho_{new}A}$.
11) Steps 3 to 10 repeat until the end of the pipe is reached. If the final pressure of the pipe does not match the state of the sphere, a new trial velocity is selected and the process begins again until the correct inlet velocity (and mass flow rate) is selected. (A search algorithm is implemented to solve this.)

I include graphs of the results for a cryostat pressure of 3.2 bar and a sphere pressure of 0.9 bar. The sharp slopes and apparent discontinuities are simply the pressure and valve pressure drops; these influence the properties of the xenon, so some sharp areas are visible in the other graphs as well.



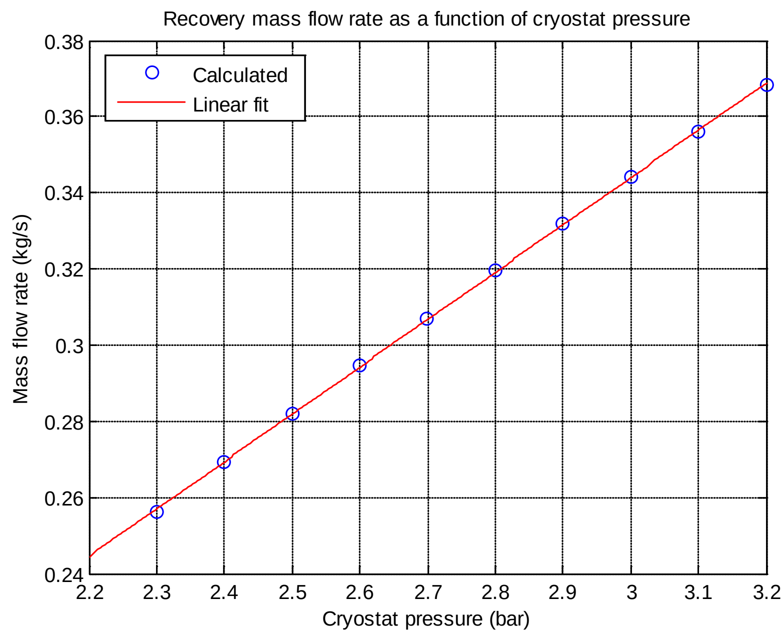


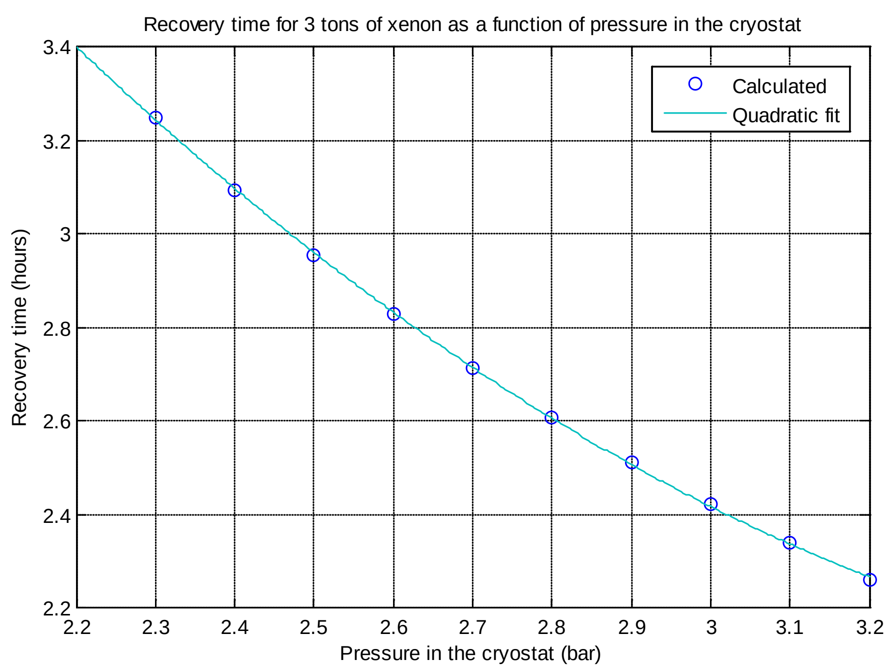




The sharp slope around the 20-meter mark in all the graphs corresponds to the valve; as you can see, it induces a sharp change in the pressure that profilerates into the other thermophysical properties. The sharp rise in the 11-meter mark for the velocity graph is due to the sudden diameter change when the cryogenics connects to the ReStoX section of the recovery line.

The important result here is that the program calculates a mass flow rate of 0.3684 kg/s at this cryostat pressure of 3.2 bar, for a recuperation time of approximately two hours and fifteen minutes. Also, the maximum velocity the xenon attains is about 27.5 m/s, which means we're well clear of any problems involving the speed of sound. Below is a plot of the flow rate as a function of the pressure in the cryostat; we are within the range of 2 hours and 15 minutes and 3 hours and 15 minutes for the entire pressure recovery range, which is fantastic news. The relationship appears to be almost precisely linear for the flow rate, so I include a fit in the plot. In addition, even though the relationship for the recovery time should be roughly inversely proportional, I include a quadratic fit for that plot as well.





If you would like a copy of the MATLAB code I used to calculate this, or have any additions or corrections you would like me to make, let me know.